Heart Dipole Tracking Algorithm

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July 8, 2007

This paper describes a method of tracking the electromagnetic dipole, with a three-dimensional representation of the dipole being graphically displayed in real-time so as to provide diagnostic information about a patient's heart. By knowing the coordinates and orientation of the heart dipole and comparing this data with a healthy heart, cardio-specialists may be able to determine where damaged tissue is located or other diseases in progress.

The method of finding the coordinates and orientation of a magnetic or electric dipole in space has eluded closed form solutions unless a large number of sensors is provided. Cirque patent number **7,145,555** describes methods of finding the location and coordinates of a magnetic dipole for use in stylus/pen devices. The equations for finding the location of an electric dipole are similar to those for finding a magnetic dipole, with the magnetic dipole moment being swapped for the electric dipole moment. The electronics for tracking an electric field are much different, however, requiring capacitive/E-field sensors which require different compensation for metal in the environment being measured, whereas magnetic sensors require compensation for permeable metals (high u metals) and the Earth's magnetic field.

A simple algorithm for locating the approximate center of the electric dipole is found by placing nine E-field sensors (sensor 1, sensor 2 and sensor 3 for discussion, each sensor set having x,y,z sensors) at a known, fixed distance away from the origin in an imaginary plane where the patient's pericardium lies as in Figure 1.

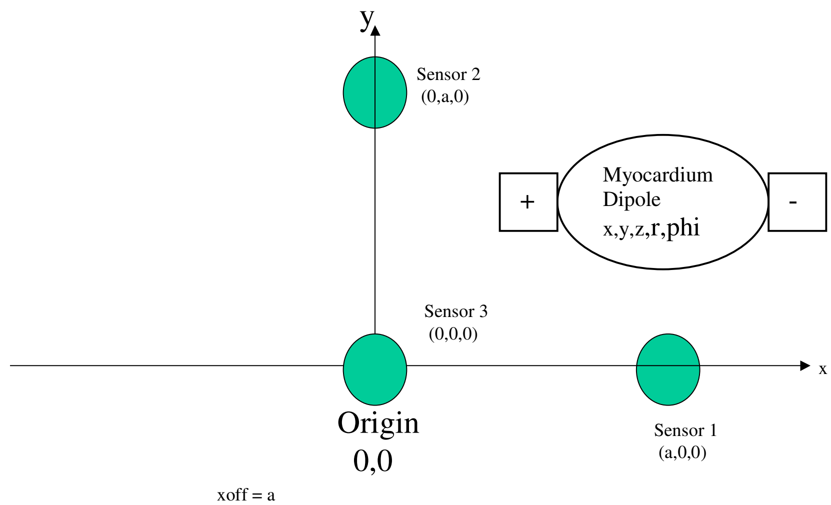

Figure 1   Sensor Placement Relative to Myocardium

The measurement from each of the three sets of field sensors will yield the following:

$$E_{mag1} = \sqrt{(E_{1x}^2 + E_{1y}^2 + E_{1z}^2}$$

$$E_{mag2} = \sqrt{(E_{2x}^2 + E_{2y}^2 + E_{2z}^2}$$

$$E_{mag3} = \sqrt{(E_{3x}^2 + E_{3y}^2 + E_{3z}^2}$$

Where E_{mag1} E_{mag2} and E_{mag3} are the magnitude of the sensor1, sensor2 and sensor3 values respectively. Then the approximate location of the center of the dipole is found by invoking the equations for the inverse-cubed law which describes how the intensity of dipoles drop in the far-field as a function of distance:

$$E_{mag1} = \frac{k_1}{r_1^3}$$

$$E_{mag2} = \frac{k_2}{r_2^3}$$

$$E_{mag3} = \frac{k_3}{r_3^3}$$

Where r is the radial vector from each sensor to the dipole centroid and r can be rewritten in terms of cartesian-coordinates x,y,z (referring to the geometry in Figure 1 and noting the value a is the distance between sensors and the origin):

$$E_{mag1} = \frac{k_1}{((x-a)^2+y^2+z^2)^{3/2}}$$

$$E_{mag2} = \frac{k_2}{(x^2+(y-a)^2+z^2)^{3/2}}$$

$$E_{mag3} = \frac{k_3}{(x^2+y^2+z^2)^{3/2}}$$

As E_{mag1}, E_{mag2}, and E_{mag3} are measured values, the equations above can be solved for x, y and z because there are three simultaneous equations in three unknowns. The solution to the equations above for x, y and z as found from MathCad is:

Given

$$E1 = \frac{k1}{\left[(x-a)^2 + y^2 + z^2\right]}$$

$$E2 = \frac{k2}{\left[x^2 + (y-a)^2 + z^2\right]}$$

$$E3 = \frac{k3}{\left(x^2 + y^2 + z^2\right)}$$

$$\text{Find}(x, y, z) \rightarrow \begin{bmatrix} \frac{1}{2} \cdot \frac{\left(E1 \cdot a^2 \cdot E3 + E1 \cdot k3 - k1 \cdot E3\right)}{E1 \cdot a \cdot E3} & \\ \frac{1}{2} \cdot \frac{\left(E2 \cdot a^2 \cdot E3 + E2 \cdot k3 - k2 \cdot E3\right)}{E2 \cdot a \cdot E3} & \\ \frac{1}{2} \cdot \frac{\left(-2 \cdot E2^2 \cdot E1^2 \cdot a^4 \cdot E3^2 + 2 \cdot E2^2 \cdot E1 \cdot a^2 \cdot E3^2 \cdot k1 - 2 \cdot E2^2 \cdot E1^2 \cdot k3^2 + 2 \cdot E2^2 \cdot E1 \cdot k3 \cdot k1 \cdot E3 - E2^2 \cdot k1^2 \cdot E3^2 + 2 \cdot E1^2 \cdot E2 \cdot a^2 \cdot E3^2 \cdot k2 + 2 \cdot E1^2 \cdot E2 \cdot k3 \cdot k2 \cdot E3 - E1^2 \cdot k2^2 \cdot E3^2\right)^{\frac{1}{2}}}{E2 \cdot a \cdot E1 \cdot E3} & \frac{-1}{2} \end{bmatrix}$$

The constants k_1, k_2 and k_3 are calibration constants that are determined by measuring the associated E_{mag1}, E_{mag2}, and E_{mag3} and the distances x , y, z relative to the pericardium with a patient who has ideal heart conditions and will be the benchmark to compare against. Constants k_1, k_2 and k_3 also include the electric dipole moment and the surrounding permittivity (epsilon) of the heart cavity. The coordinates x,y,z found relative to the sensor array place the location of the centroid of the heart dipole, which when the direction and angle shifts relative to the ideal heart dipole location, should provide data useful in diagnosis of heart conditions. Because of the three-dimensional nature of the algorithm, the formulations above work as well for sensors that lie in an plane parallel to the plane of the pericardium such as in Figure 1.

For a complete solution of the dipole coordinates that works in the near as well as the far field, we turn to the electric scalar potential Φ:

$$\Phi = \frac{\mathrm{p}\cos(\theta)}{r^2}$$

Where the divergence of the scalar potential holds:

$$\nabla\Phi = -\vec{E}$$

And the E vectors of the dipole in spherical coordinates relative to the reference frame of the dipole is found to be:

$$E_r = \frac{p\cos(\theta)}{r^3}$$

$$E_\theta = \frac{p\sin(\theta)}{r^3}$$

$$E_\phi = 0$$

Assuming the dipole reference frame is parallel to the sensor reference frame, then the equations for the measured values for sensor 3 would be:

$$E_{1x} = E_r\cos(\theta)\sin(\phi) + E_\phi\cos(\theta)\cos(\phi)$$
$$E_{1y} = E_r\cos(\theta)\cos(\phi) + E_\phi\cos(\theta)\sin(\phi)$$
$$E_{1z} = E_r\sin(\theta) - E_\phi\cos(\theta)$$

Similar equations can be written for sensor 2 and sensor 3 which translate the reference frame of the dipole to the reference frame of the sensors (in fact, the equations relative to the dipole reference frame can be translated using quaternions, a standard translation technique). The equations that result for each set of x,y,z sensors at sensor locations 1, 2 and 3 produces nine equations in six unknowns (x, y, z, p, θ, ϕ) which results in an overdetermined (and easier to solve) solution. The angles θ, ϕ are new information that give the orientation of the dipole relative to the sensor array. The original coordinates (x,y,z) are also useful along with the orientation, to determine the beginning and endpoints of the dipole in 3-space. The value p is the electric dipole moment which is determined as a result of the equations, and this eliminates the need for calibration as mentioned in the previously simpler method of location. The determination of p for each patient allows complete independence between ideal and individual conditions, requiring no calibration of the sensor array in the manufacturing environment.

The methods described allow for an approximate or complete solution of the location and orientation of the electric dipole in the myocardium.